**Anti-windup Observer-based Controller Using Improved Bounded Real Lemma: Active Vibration Control**

**Abstract**

In this paper, the windup problem is addressed for disturbance rejection control (DRC) based on an unknown input observer (UIO). The proposed method aims at dealing with actuator input nonlinearities in active vibration control (AVC) to provide a useful tool for systems under unknown environmental mechanical stimuli. Accordingly, first, assuming the nonlinear governing dynamics of the plant to be smooth and differentiable around the equilibrium points, a UIO is designed such that the two transfer functions from disturbance/control input to observation residuals are null. By aiming at rejecting the external perturbations in the plant output and the observation error dynamics, an improved form of the bounded real lemma is employed to guarantee the quadratic stability. Then, based on the Lipschitz condition on nonlinear terms and considering independent input disturbance signal (mismatch disturbance) and measurement noise, DRC synthesis is formulated in linear matrix equality/inequality framework. To address the saturation of actuators, the developed controller is then augmented with a static anti-windup (AW) compensator in a linear fractional transformation (LFT) form. The proposed method can be used as a low order yet effective tool in model-based AVC of arbitrary mechanical/civil structures.

**Keywords:** bounded real lemma; linear matrix inequality; actuator saturation; observer-based control; active vibration control

1. **Introduction**

In the last two decades, by aiming at high precision process control, various candidates are proposed for estimation of the states of the system (Ciccarella *et al*., 1993; Raghavan and Hedrick, 1994; Rajamani, 1998, Yaz and Yaz, 2001; Ha and Trinh, 2004; Lu *et al*., 2006; Mondal *et al*., 2010; Oveisi and Nestorović, 2016a). The structure of the estimator is an essential design decision that determines the dynamics of state estimation error. As a result, the estimation tools such as Kalman filter, high gain observer, and more recent observers developed in linear matrix inequality (LMI) framework may have different behavior when the system is subjected to input/output time-varying disturbances and measurement noise (Darouach et al., 1994; Koenig and Mammar, 2001; Pertew et al., 2005; Koenig, 2006; Xu, 2008). In the present research, by aiming at providing a simple yet effective tool for AVC of mechanical/civil structures with actuator saturation, DRC based on a UIO is augmented with static AW compensator. In literature, the mismatch DRC with AW compensator for AVC is not tackled with some exceptions (Zaccarian and Teel, 2000; Teel et al., 2006). The augmented control design is proposed in terms of a two-step convex feasibility problem with linear matrix equality (LME) constraints. Accordingly, first, a UIO is utilized to reconstruct the states of the system under unknown but bounded disturbance and noise signals following a similar approach as Mondal *et al*. (2008; 2010). It is assumed that the nonlinear governing equation of motion for AVC purposes satisfies the Lipschitz condition. Compared to Kalman filter, in which the state and control input matrices in estimator dynamics, are exact replicas of the nominal model, in the proposed method, a less conservative model is initiated and then additional constraints are introduced on estimator parameters/structure in terms of some LMEs. In order to keep the feedback control design problem linear, these LME constraints are preliminary solved before moving to DRC based on quadratic stability lemma. As an advantage, the latter method has a better performance mainly because the transfer function from disturbance to the states observation residual is set to null. Then, the observed states are utilized to form a feedback signal for coupled DRC synthesis. For this purpose, the bounded real lemma (BRL) is presented as a proposition and then, an extended BRL proposed to reject the disturbance at the plant output quadratically.

Since the control effort may pass through nonlinear functions that are not sufficiently smooth to be considered in the form of Rajamani (1998), such as actuator saturation, first, they are neglected for observer/controller design. These nonlinearities are then later handled by the anti-windup bumpless transfer (AWBT) compensator (Kothare and Morari, 1997 & 1999; Mulder et al., 2001; Hu et al., 2006; Tarbouriech and Turner, 2009; Wu and Lin, 2014). In this paper, such nonlinearities will be addressed employing the absolute stability theorem (Mulder et al., 2001). Finally, the main results of the paper as a framework for AVC of civil/mechanical structures is presented and later is implemented on a mechanical vibrating system (Zaccarian and Teel, 2000; Teel et al., 2006; Oveisi and Nestorović, 2016b).

In the rest of the paper, represents the identity matrix with appropriate dimension, superscript signifies the transpose of a matrix or a vector, and stands for the set of real numbers. Additionally, , , and represent time dependent vectors, matrices, and positive definite matrices with dimensions of , , for , respectively. Also, represents the expected value of a random variable.

1. **Problem formulation**

Consider the multi-input/multi-output nonlinear dynamics of a plant in the form of Eq. (1)

|  |  |
| --- | --- |
|  | (1) |

where, ,,, and are the state, input, square-integrable disturbance, white Gaussian noise, and output vectors at time , respectively. In addition, in order to put the system in the class of nonlinear Lipschitz systems, it is assumed that , , , and are sufficiently smooth and differentiable for UIO/DRC synthesis. Then, the necessary and sufficient conditions on the asymptotic stability of the observer-based control system can be addressed by the eigenvalues of the linearized system state matrix (Rajamani, 1998) (see Eq. (2)). For causal system in Eq. (1), with globally Lipschitz continuous dynamics, Eq. (1) can be linearized in the form of Eq. (2)

|  |  |
| --- | --- |
|  | (2) |

where , , , , and assuming that to keep the design problem of the observer-based controller convex. In Eq. (2) and the rest of the paper, for simplifying the notation, the dependence of the system terms on time is suppressed. Assuming that *i*) is asymptotically stable, *ii*) is detectable, *iii*) matrices and are full row and column rank, correspondingly, and *iv*) has the same rank as , the dynamics of the UIO mechanism is presented in Eq. (3).

|  |  |
| --- | --- |
|  | (3) |

In Eq. (3), , , and are the observed state-, output-, and auxiliary observer state vectors, correspondingly.

**Definition 1.** The state reconstruction system based on UIO in Eq. (3) is asymptotically stable if *a*) The Lyapunov stability of the estimator is satisfied and *b*) if the matrices , , , and can be calculated such that .

The observation residual is defined as . Then, using Eqs. (2, 3), the dynamics of state-estimation error can be derived as Eq. (4)

|  |  |
| --- | --- |
|  | (4) |

where and . Following (Mondal *et al*., 2010), in order to cancel the direct effects of the states, control input, and input disturbance on the error dynamics it is assumed that

|  |  |
| --- | --- |
|  | (5a) |
|  | (5b) |
|  | (5c) |

Then, employing Eq. (5c), we obtain , where and with being a generalized inversion. If , there exists a unique solution for the intermediate matrix that guarantees the satisfaction of Eq. (5c). In such case, can be calculated numerically. If , then a solution for may be calculated by means of Least Square (LS) algorithm. Next, by using Eq. (5c) and after some mathematical manipulations, and are calculated as and , where and . Finally, by using Eq. (5a) and introducing , the states estimation error dynamics can be obtained as . Since, the ultimate goal of the paper is to develop an observer-based DRC, the control law is defined as .

**Proposition 1.** (BRL) (Ahmadizadeh *et al*., 2014) *System and is asymptotically stable if there exists a symmetric positive definite matrix and a positive scalar such that the following LMI is satisfied*

|  |  |
| --- | --- |
|  | (6) |

**Proposition 2.** (Improved BRL) *System , and is asymptotically stable and the quadratic stability of the transfer function from the input disturbance () and measurement noise () to the system output is guaranteed if there exist symmetric positive definite matrices , , and and positive scalars and such that the following LMI is satisfied*

|  |  |
| --- | --- |
|  | (7) |

*where and represents the transpose of all upper triangular elements of*

*Proof*. Select the Lyapunov function as . Then, derivation of the Lyapunov function with respect to time results in in which . This completes the proof.

■

**Theorem 1.** *System (2) together with the state estimation mechanism (3) is asymptotically stable in terms of Definition 1 and rejects the input disturbance and measurement noise in output quadratically if there exist symmetric positive definite matrices and , matrices , , and , positive scalars , , and , and user-defined positive weighting scalars and such that LMI condition (8) is satisfied*

|  |  |
| --- | --- |
|  | (8a) |

*where,*

|  |  |
| --- | --- |
|  | (8b) |

*Then, the controller gain , and observer gains and can be calculated*.

*Proof*. The proof is a direct use of Proposition 2 and Eq. (7) on the Lyapunov function to obtain in which . Then, setting and and applying the congruence transformation on and defining , can be obtained. This completes the proof.

■

Using Theorem 1, the extended observer-based controller dynamics can be transformed in Eq. (9)

|  |  |
| --- | --- |
|  | (9) |

where , , , and .

Based on Kothare *et al*. (1994), the closed-loop system is configured in axiomatic compensation framework for the regulation problem presented in Fig. 1a. In this figure, the augmented controller is parameterized by system matrices , also, , , and represents actuation nonlinearity (saturation function in this paper), AW compensation signal, and a static gain matrix for AW compensation, respectively. The augmented controller’s configuration guarantees the independent effect of the AW compensator on controller states and output (Mulder *et al*., 2001).

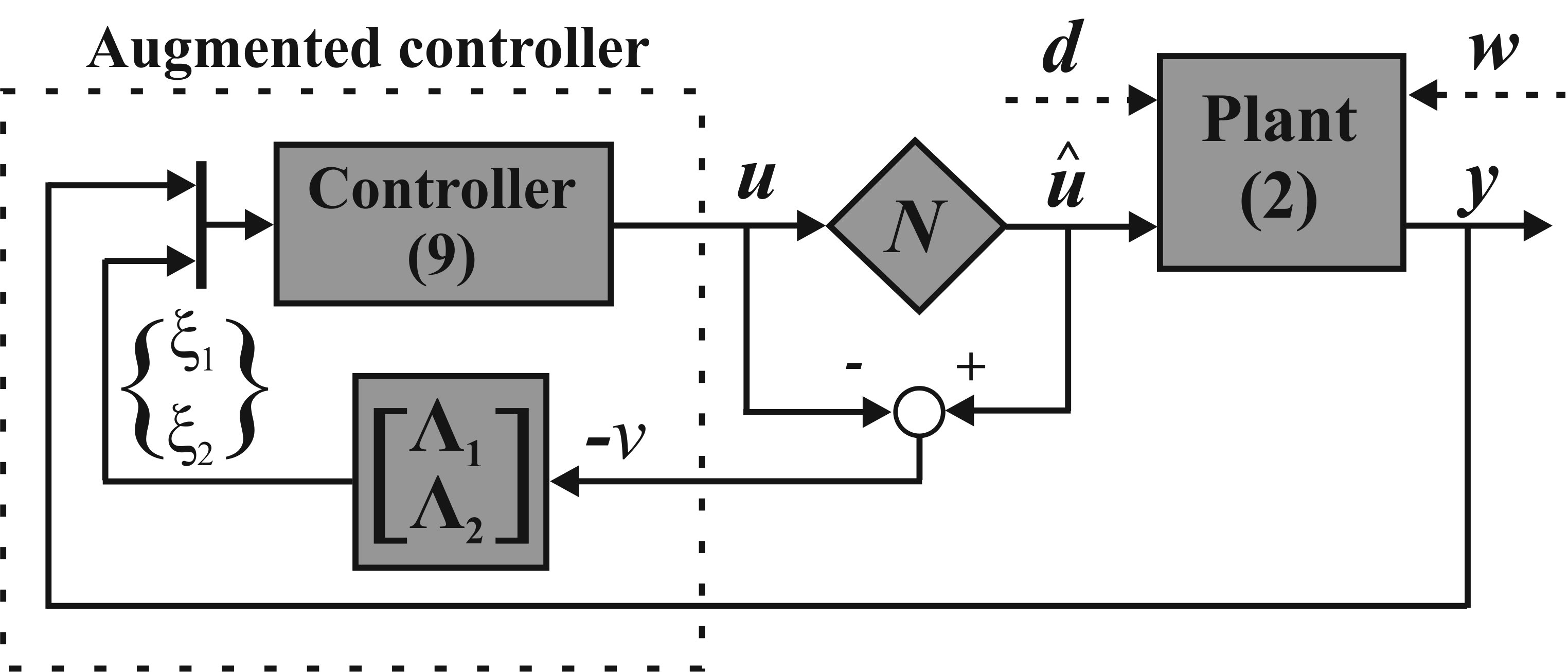


Fig. 1 The AW observer-based closed-loop system interconnection

Marcopoli and Philips (1996) and Mulder *et al*. (1999) presented an apparent convexity in controller synthesis for plants with the saturated actuators. Their observation led to recasting the AW synthesis in LMI framework by transforming the system in Fig. 1a to LFT form of Fig. 2. In this figure, all of the exogenous inputs are included in and . As a natural claim on the AW system, when , the controller recovers from Fig. 1 to linear observed state-feedback controller obtained from Theorem 1. The dynamic equation governing in Fig. 2 can be formulated as (Mulder *et al*., 2009)

|  |  |
| --- | --- |
|  | (10a) |
|  | (10b) |

where,

|  |  |
| --- | --- |
|  | (10c) |

Additionally, , , , , , , , , , , , are matrices with appropriate dimension that can be obtained from minimal state-space realization of Eq. (10b). By reducing Fig. 1a in to LFT form in Fig1b, the objective of static AW compensator is to design .

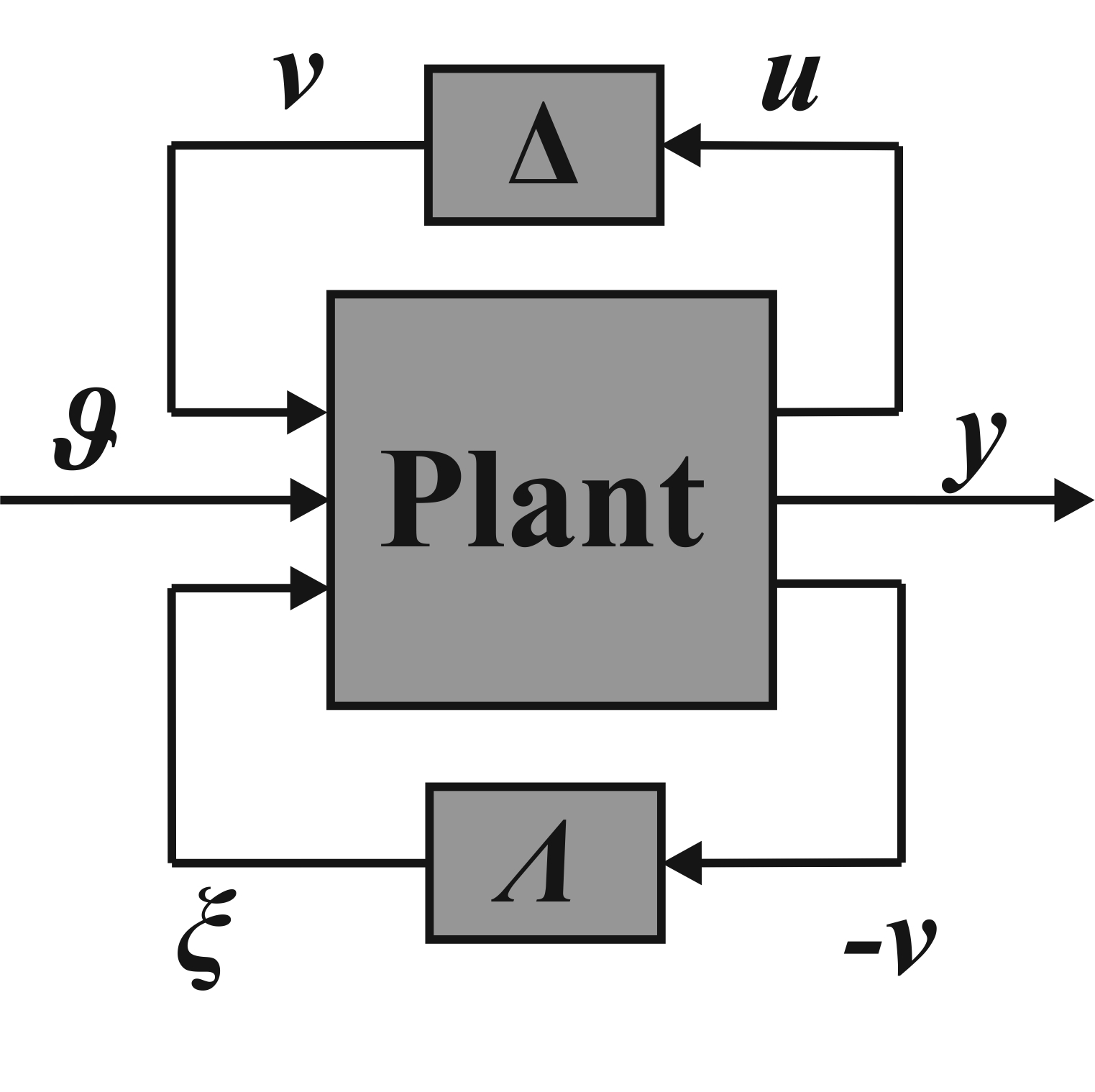


Fig. 2 Standard AW LFT configuration

**Theorem 2.** ( gain criterion) [Teel and Kapoor, 1997; Mulder *et al*., 2001] *The AW compensator in Eq. (10) is gain stabilizable, if there exists symmetric positive definite matrix with appropriate dimension, positive definite block diagonal matrix , matrix and positive scalars , , and such that the LMI equation (11) is satisfied*

|  |  |
| --- | --- |
|  | (11a) |

*with*

|  |  |
| --- | --- |
|  | (11b) |

*Then, .*

1. **Numerical example**
   1. **Modeling of open loop plant**

In this section, the unknown input observer-based DRC augmented with AW compensator is implemented on a smart structure which is oscillating under the effect of a bounded disturbance signal. The objective of AWDRC is to attenuate the vibration of the structure at the system output. Additionally, if the saturation appears in the actuation elements, the AW compensator accounts for the matter to keep the stability of the system and if , the controller should recover from wind-up state. The plant configuration, as shown in Fig. 3, consists of a clamped-free aluminum beam with Young’s modulus 70 Gpa and density 2.7 g/cm³ and two piezo-patches ( mm) bounded on one side of the structure. The piezo-actuators (DuraActTM P-876.A15) are activated in operating voltage range of [-150 150] V with electrical capactitance of 45 nF, and blocking force of 775 N.

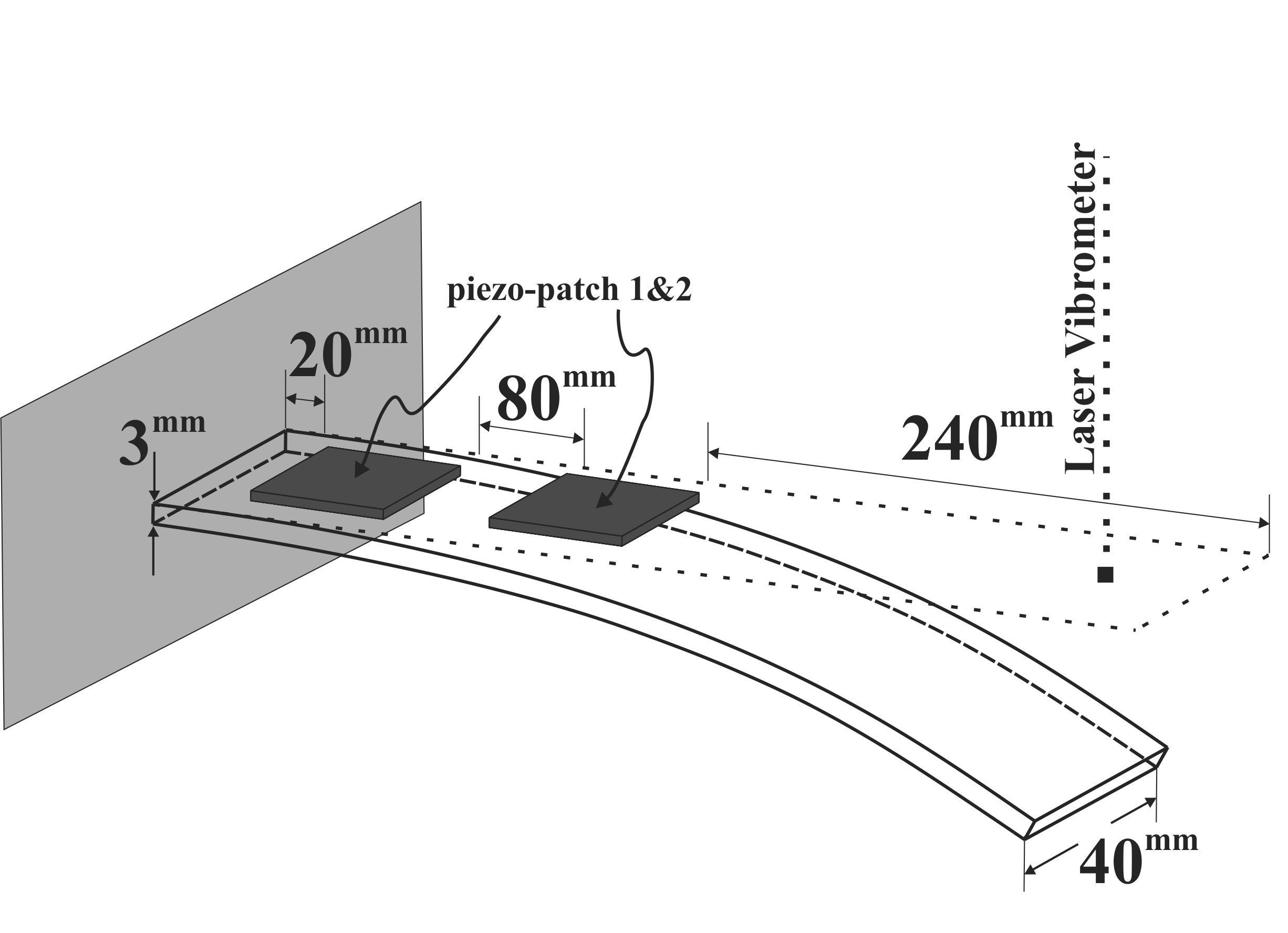


Fig. 3 Geometry of the smart beam

The closed-loop system configuration is presented in Fig. 4. The piezolaminated beam plant has three independent inputs: the control inputs which act on the two actuator piezo-patches and the exogenous signal which excites the structure through an independent disturbance channel.

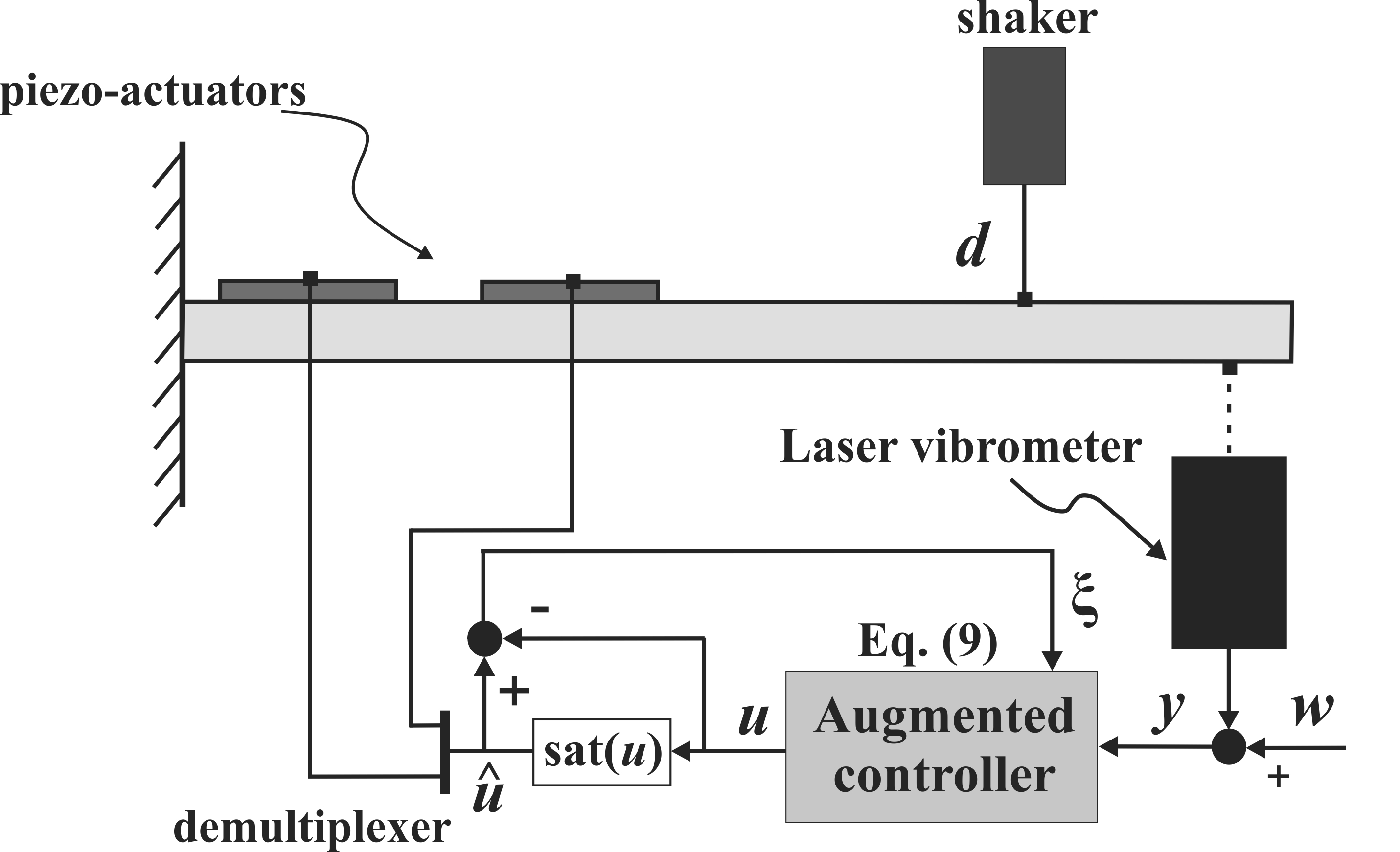


Fig. 4 Schematic interconnection of the plant and extended observer-based AW-controller

It should be mentioned that the torsional modes are neglected in the modeling of the beam since only the bending vibration suppression is analyzed in this paper. The dynamics of the actuation is obtained using finite element analysis in coupled electro-mechanical domain. By assuming , , and as mass, damping, and stiffness matrices in the framework of FEM, the governing ordinary differential equation (ODE) of motion in matrix form can be written as with and being the nodal states of displacement and electric potential/applied external inputs, respectively (Oveisi and Nestorović, 2014). For control purposes, the structure of control effort is assumed to be , where, represents the position of generalized control effort in FE structure. The output of the plant based on the nodal displacements and velocities is expressed as with and being the modal output displacement and velocity matrices, respectively. The natural frequencies () of the coupled structure as well as the mode shapes () for are obtained by employing the classical harmonic solution () and solving the obtained algebraic equation. Then, by applying the transformation , the nodal model of the plant can be transformed to the generalized modal displacement model, in which, is the matrix of eigen vectors. Finally, by using the proportional damping model, , orthogonality of mode shapes, and assuming the state vector as , with , the governing equation of motion can be presented in state space form of Eq. (2). The interpretation of the state, input, and output matrices based on the FE approach can be presented as Eq. (12) (Marinkovic *et al*., 2009)

|  |  |
| --- | --- |
|  | (12) |

where with being the damping ratio of th mode shape and , , and **.**

* 1. **Controller and Observer design**

In this section, the numerical results of the employment of the proposed controller are presented on the piezo-laminated beam shown in Fig. 3. For design purposes, a reduced order system is obtained by considering two bending mode shapes of the smart structure. The two fundamental natural frequencies of the system are 87.3 and 480 rad/s and their associated damping ratio is assumed to be 1. The nominal system is a transformed in state space model with the system matrices shown in Eq. (13).

|  |  |
| --- | --- |
|  | (13) |

For examination of the enactment of the closed-loop system, an observer-based controller is designed using Theorem 1. First, the controller () and observer system (Eq. 3) are designed based on the results in Theorem 1. Accordingly, the controller and observer gains are obtained as

|  |  |
| --- | --- |
|  | (14a) |

The results in Eq. (14a) are obtained based on the intermediate parameter in Eq. (14b)

|  |  |
| --- | --- |
|  | (14b) |

Next, by transforming the results in Eq. (14) into Eq. (9) and then (10), and finally utilizing Theorem 2, the AW static compensation gain is calculated in Eq. (15)

|  |  |
| --- | --- |
|  | (15) |

By implementing the control system based on matrices in Eqs. (14, 15) on the plant with system matrices as in Eq. (13), the prospect of the operational vibration attenuation performance is assessed. For this purpose, first, the disturbance channel is activated by the realization of a chirp signal. The frequency of the interference signal is swept from zero to 90 Hz within a nine seconds time window in three cases of the AWDRC, saturated DRC, and uncontrolled systems. Investigations are carried out in the time domain by means of the structure in Fig. 3 with the sampling frequency of 10 kHz. All of the control systems, as well as the plant, are modeled on SIMULINK platform with standard fixed step explicit ODE5 solver (Dormand-Prince method)[[1]](#footnote-1). The response of the system for the closed-loop and open loop cases is shown in Fig. 5a in the time domain based on the velocity measurement signal.

|  |  |
| --- | --- |
|  |  |
| Fig. 5a Comparison of measured outputs: performance of the closed-loopsystem in saturated case | Fig. 5b Comparison of measured outputs in time domain: AW performance reduction |

It can be seen in Fig. 5a that the control system based on Theorem 1 is unable to deal with actuation nonlinearity and although the AWDRC lost performance compared to the unsaturated system, it suppressed the vibration magnitude within the considered frequency range. Moreover, the hybrid compensator based on Theorems 1 and 2 is asymptotically stable that can recover from the windup state. This can be seen by comparing the control effort in Fig. 6a with the system output Fig. 5b at region 1 in which the performance of the AW compensated controller reaches to linear case. Additionally, Fig. 6b represents the control effort for the unsaturated system.

|  |  |
| --- | --- |
|  |  |
| Fig. 6a control effort: AW-compensated obsever-based controller | Fig. 6b control effort: obsever-based controller (Theorem 1) |

Next, the residual of the observation of the system output is shown in Figs. 7 for three cases: AWDRC, unsaturated DRC, and estimation error based on Kalman filter. For this purpose, first, for the nominal deterministic model (Eq. 2), the well-known Kalman filter is calculated by finding the solution of the algebraic Riccati equation (ARE): . In this equation, with , , and is the steady-state error covariance (). Consequently, the obtained is presented in Eq. (16).

|  |  |
| --- | --- |
|  | (16) |



Fig. 7. Estimation error of the closed-loop system: comparison between AWDRC and unsaturated DRC

It can be seen that the proposed observer method has a better performance in state observation than the Kalman observer. Finally, in order to examine the disturbance rejection performance of the proposed method in Theorem 1, it is compared to the Linear Quadratic Gaussian (LQG) controller in combination with AW compensator in Theorem 2. The comparison of the vibration suppression quality is presented in Fig. 8. It can be seen that the DRC based on Theorem 1 has a better performance compared to AW-compensated LQG controller. The control effort for both of the closed-loop systems, AWDRC, and AWLQG, under the external disturbance, is depicted in Fig. 9a and 9b, respectively.



Fig. 8 Comparison of measured outputs: instability of closed-loop in saturated case

|  |  |
| --- | --- |
|  |  |
| Fig. 9a Control effort: AWDRC | Fig. 9b Control effort: AWLQG |

**Conclusion**

In this paper, the trend in unknown input observation is used to estimate the states of the system in a less conservative agenda. For this purpose, a set of LMEs is defined to account for multiple unknown matrices in dynamics of the observer. Then, the remaining parameters are acquired as a solution of coupled asymptotic stability problem in terms of improved BRL and stability of the observation residuals. The solution of the observer-based controller is transformed in the framework of static AWBT compensation. For this purpose, the absolute stability of the coupled system is guaranteed by solving and additional LMI. Both of the proposed observer and controller are compared with standard Kalman Filter and LQG regulator, respectively, on a mechanical vibrating plant. It has been observed that the proposed observer-based method has a better performance in vibration attenuation. Additionally, for nonlinear systems with multiple equilibrium points, the proposed method can be easily generalized by employing the Takagi-Sugeno fuzzy global approximation (Tanaka and Wang, 2001).

**References**

1. Ciccarella, G., Dalla Mora, M., Germani, A. (1993). A Luenberger-like observer for nonlinear systems. *International Journal of Control* 57(3):537-556.
2. Raghavan, S., Hedrick, J. K. (1994). Observer design for a class of nonlinear systems. *International Journal of Control* 59(2):515-528.
3. Rajamani, R. (1998). Observer for Lipschitz nonlinear systems. *IEEE Transactions on Automatic Control* 43(3):397-401.
4. Yaz, E. E., Yaz, Y. I. (2001). LMI based observer design for nonlinear systems with integral quadratic constraints. *40th IEEE Conference on Decision and Control*, Orlando, USA, 2954-2955.
5. Ha, Q. P., Trinh, H. (2004). State and input simultaneous estimation for a class of nonlinear systems. *Automatica* 40(10):1779-1785.
6. Lu, J., Feng, C., Xu, S., Chu, Y. (2006). Observer design for a class of uncertain state-delayed nonlinear systems. *International Journal of Control, Automation and Systems* 4(4):448-455.
7. Mondal, S., Chakraborty, G., Kingshook, B. (2010). LMI approach to robust unknown input observer design for continuous systems with noise and uncertainties. *International Journal of Control, Automation, and Systems* 8(2):210-219.
8. Oveisi, A., Nestorović, T. (2016). Robust observer-based adaptive fuzzy sliding mode controller. *Mechanical Systems and Signal Processing* 76-77(2016):58-71.
9. Darouach, M., Zasadzinski, M., Xu, S. J. (1994) Full order observers for Linear systems with unknown inputs. *IEEE Transactions on Automatic Control* 39(3):606-609.
10. Koenig D., Mammar, S. (2001). Design of a class of reduced order unknown inputs nonlinear observer for fault diagnosis. *Proceedings of the 2001 American Control Conference*, USA 2143-2147.
11. Pertew, A. M., Marquez, H. J., Zhao, Q. (2005). H∞- synthesis of unknown input observers for non-linear Lipschitz systems. *International Journal of Control* 78(15):1155-1165.
12. Koenig, D. (2006). Observer design for unknown input nonlinear descriptor systems via convex optimization. *IEEE Transactions on Automatic Control* 51(6):1047-1052.
13. Xu, J., Sun, M., Yun, L. (2008). LMI-based synthesis of robust iterative learning controller with current feedback for linear uncertain systems. *International Journal of Control, Automation and Systems* 6(2):171-179.
14. Kothare, M. V., Morari, M. (1997). Multivariable anti-windup controller synthesis using multi-objective optimization. *Proceedings of the 1997 American Control Conference*, Albuquerque, New Mexico 3093-3097.
15. Kothare, M. V., & Morari, M. (1999). Multiplier theory for stability analysis of anti-windup control systems. *Automatica* 35(5):917-928.
16. Mulder, E. F., Kothare, M. V., Morari, M. (2001). Multivariable anti-windup controller synthesis using linear matrix inequalities. *Automatica* 37(2001):1407-1416
17. Hu, T., Teel, A. R., Zaccarian L. (2006). Stability and performance for saturated systems via quadratic and nonquadratic Lyapunov functions. *IEEE Transactions on Automatic Control* 51(11):1770 – 1786.
18. Tarbouriech, S., Turner, M. (2009). Anti-windup design: an overview of some recent advances and open problems. *IET Control Theory & Applications* 3(1):1-19.
19. Wu, X., Lin, Z. (2014). Dynamic anti-windup design in anticipation of actuator saturation. *International Journal of Robust and Nonlinear Control* 24(2):295-312.
20. Zaccarian, L., Teel, A. R. (2000). A benchmark example for anti-windup synthesis in active vibration isolation tasks and an L2 anti-windup solution. *European Journal of Control* 6(5):405-420.
21. Teel, A. R., Zaccarianb, L., Marcinkowski J. J. (2006). An anti-windup strategy for active vibration isolation systems. *Control Engineering Practice* 14(1):17-27.
22. Oveisi, A., Nestorović T. (2016). Mu-synthesis based active robust vibration control of an MRI inlet. *Facta Universitatis, Series: Mechanical Engineering* 14(1):37-53
23. Oveisi, A., Shakeri, R. (2016). Robust reliable control in vibration suppression of sandwich circular plates. *Engineering Structures* 116(2016): 1-11.
24. Ahmadizadeh, S., Zarei, J., Karimi H. R. (2014). Robust unknown input observer design for linear uncertain time delay systems with application to fault detection. *Asian Journal of Control* 16(4):1006-1019.
25. Kothare, M. V., Campo, P. J., Morari, M., Nett, C. N. (1994). A unified framework for the study of anti-windup designs. *Automatica* 30(12):1869-1883.
26. Marcopoli, V. R., Phillips, S. M. (1996). Analysis and synthesis tools for a class of actuator-limited multivariable control systems: a linear matrix inequality approach. *International Journal of Robust and Nonlinear Control* 6(9-10):1045-1063.
27. Mulder, E. F., Kothare, M. V., Morari, M. (1999). Multivariable anti-windup controller synthesis using iterative linear matrix inequalities. *Proceedings of the 1999 European Control Conference*, Karlsruhe, Germany.
28. Mulder, E. F., Tiwarib, P. Y., Kothare, M. V. (2009). Simultaneous linear and anti-windup controller synthesis using multi-objective convex optimization. *Automatica* 45(3):805-811.
29. Teel, A. R., Kapoor, N. (1997). The L2 anti-windup problem: Its definition and solution. *Proceedings of the 1997 European Control Conference*, Brussels, Belgium.
30. Oveisi A., Nestorović T. (2014). Robust mixed H2/H8 active vibration controller in attenuation of smart beam. *Facta Universitatis, Series: Mechanical Engineering* 12(3):235-249.
31. Marinkovic, D., Koppe. H., Gabbert, U. (2009). Aspects of modeling piezoelectric active thin-walled structures. *Journal of Intelligent Material Systems and Structures* 20(15):1835-1844.
32. Tanaka, K., Wang, H. O. (2001). Fuzzy *Control Systems Design and Analysis: A Linear Matrix Inequality Approach.* Wiley-Interscience.

1. Authors are willing to share the script for designing the controller/observer as an .m file as well as the SIMULINK model with the interested reader. [↑](#footnote-ref-1)